DYNAMIC STABILITY OF MULTILAYERED BEAM WITH BOLTED JOINTS

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology
in
Mechanical Engineering
By

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National Institute of Technology
Rourkela

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CERTIFICATE

This is to certify the thesis entitled, "**Dynamic Stability of Multilayered Beam With Bolted Joints**" submitted by **Sri Sadikababa Shaik** in partial fulfillment of the requirements for the award of **Master of Technology in Mechanical Engineering** with specialization in "**Machine Design and Analysis**" at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date

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(Sadikababa shaik)

CONTENTS

ABSTRACT

Dynamic stability of elastic systems deals with the study of vibrations induced by pulsating load that are parametric with respect to certain form of deformations. An important aspect in the analysis of parametric dynamic system is the establishment of the regions in the parametric space in which the system in unstable, these regions are known as regions of dynamic instability.

The boundary separating the stable zones from unstable zones is called stability boundaries and plot of these boundaries on the parametric space is called a stability diagram. Beams are the simplest structural members in any machine, mechanism or large structure. Layered structures are gaining importance in present day use because of their good vibration damping characteristics.

The objective of the present work is to study the dynamic stability of a multilayered bolt jointed beam with fixed-fixed boundary condition and subjected to a pulsating end force. Finite element method is used for theoretical analysis. Experiments have been carried out to validate the theoretical findings. For this a test rig has been designed and fabricated to provide the required variable loading and necessary attachments have also been fabricated to simulate the fixed end conditions.

The theoretical and experimental investigations show that the multilayered bolt jointed beam with fixed-fixed end condition is more stable than a single layered beam with similar end condition. The stability of the beam improves with increase in number of layers. The theoretical results matched very well with the experimental one.

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NOMENCLATURE

A	Cross-sectional area of the element
[C]	Damping matrix
E	Modulus of elasticity
I(X)	Moment of inertia
[K]	Stiffness matrix
$[K_e]$	Element stiffness matrix
$[K_G]$	Element geometric stiffness matrix
l	Element length
[M]	Mass matrix
P(t)	Axial periodic load
P_t	Time dependent component of the periodic load
P_S	Static component of the periodic load
{q}	Assemblage nodal displacement vector
[S]	Stability matrix
$[S]^e$	Element Stability matrix
t	Variable time
T	Kinetic energy
u	Axial displacement of node
V	Potential energy
v	Transverse displacement of node
x	Coordinate along the axis of the beam
α	Static load factor
β	Dynamic load factor
ρ	Mass density
Ω	Disturbing frequency
ω	Fundamental natural frequency
μ	Damping coefficient
ξ	Damping factor

CHAPTER 1

INTRODUCTION

INTRODUCTION

1.1 Introduction:

The theory of the stability of elastic system deals with the study of vibration induced by pulsating force that are parametric with respect to certain form of deformations. A system is said to be parametrically excited if the time dependent frequency of the excitation appears as one of the variable coefficients of homogeneous differential equation describing the system, unlike external excitation, which leads to an in homogeneous differential equation. Well-known form of equation describing a parametric system is Hill's equation.

$$[M]\left\{\ddot{q}\right\}+[C]\left[\dot{q}\right]+\left[[K_e]-P(t)[S]\right]\{q\}=0 \tag{1.1}$$

When $P(t) = \cos\Omega$ t,

Equation (1.1) is known as Mathieu's equation.

Equation (1.1) governs the response of many systems to a sinusoidal parametric excitation. In practical, parametric excitation can occur in structural system subjected to vertical ground motion, aircraft structures subjected to turbulent flow, and in machine components and mechanisms. Other examples are longitudinal excitation of rocket tanks and their liquid propellant by the combustion chamber during powered flight, helicopter blades in forward flight in a free-stream that varies periodically and spinning satellites in elliptic orbits passing through a periodically varying gravitation field.

In industrial machine and mechanisms, their components and instruments are frequently subjected to periodic or random excitation transmitted through elastic coupling elements, example includes those associated with electromagnetic, aeronautical, vibratory conveyors saw blades and belt drives.

In parametric instability the rate of increase in amplitude is generally exponential and thus potentially dangerous in typical resonance the rate of increase is linear. Moreover damping reduces the severity of typical resonance, but may only reduce the rate of increase during parametric resonance. Moreover parametric instability occurs over a region of parametric space and not at discrete points. The apparently simple second order differential equation (1.1) has no exact analytical solution and thus the solution may have very complicated behaviors, because of the coefficients being time dependent.

The system can experience parametric instability (resonance) when the excitation frequency or any integer multiple of it is twice the natural frequency that is to say

$$m\Omega = 2\omega_n \text{, m=1, 2, 3,.................................. } n$$

The case $\Omega = 2\omega_n$ is known to be the most important in application and is called main parametric resonance. A vital step in the analysis of parametric instability is the establishment of zones of parametric resonance. The boundary separating a stable region from an unstable one is called a stability boundary. Plot of these boundaries on the parameter space is called a stability diagram.

CHAPTER 2

LITERATURE REVIEW

LITERATURE REVIEW

2.1 INTRODUCTION

There have been a lot of research works on the dynamic stability of beams during the last two decades. The present literature review is restricted only to the study of methods of stability analysis of parametrically excited system, types of parametric resonance, effects of damping, and effects of bolted joints

2.2 METHODS OF STABILITY ANALYSIS OF PARAMETRICALLY EXCITED SYSTEM

The governing equations for parametrically excited systems are second order differential equations with periodic coefficients, which have no exact solutions. The researchers for a long time have been interested to explore different solution methods to this class of problem. The two main objectives of this class of researchers are to establish the existence of periodic solutions and their stability. When the governing equation of motion for the system is of Matheiu-Hill type, a few well known solution methods those are commonly used are, method proposed by Bolotin based on Floquet's theory, perturbation and iteration techniques, the Galerkin's method, the Lyapunov second method and the asymptotic technique by Krylov, Bogoliubov and Mitroploskii. Bolotin's [16] method based on Floquet's theory can be used to get satisfactory results for simple resonance only. Steven [31] later modified the Bolotin's method for system with complex differentials equation of motion. Hsu [20-21] proposed an approximate method of stability analysis of systems having small parameter excitations.

Hsu's method can be used to obtain instability zones of main, combination and difference types. Later Saito and Otomi [27] modified Hsu's method to suit systems with complex differential equation of motion. Takahashi [33] proposed a method free from the limitations of small parameter assumption. This method establishes both the simple and combination type instability zones. Zajaczkowski and Lipinski [35] and Zajaczkowski [36] based on Bolotin's method derived formulae to establish the regions of instability and to calculate the steady state response of systems described by a set of linear differential equations with time dependent parameters represented by a trigonometric series. Lau et al. [25] proposed a variable parameter incrementation method, which is free from limitations of small excitation parameters. It has the advantage of treating non-linear systems.

Many investigators to study the dynamic stability of elastic systems have also applied finite element method. Brown et al. [17] studied the dynamic stability of uniform bars by applying this method. Abbas [13] studied the effect of rotational speed and root flexibility on the stability of a rotating Timoshenko beam by finite element method. Abbas and Thomas [12] and Yokoyama [34] used finite element method to study the effect of support condition on the dynamic stability of Timoshenko beams. Shastry and Rao [28] by finite element method obtained critical frequencies and the stability boundaries [29-30] for a cantilever column under an intermediate periodic concentrated load for various load positions. Bauchau and Hong [14] studied the non-linear response and stability of beams using finite element in time. Briseghella et al. [15] studied the dynamic stability problems of beams and frames by using finite element method. Svensson [32] by this method studied the stability properties of a periodically loaded non-linear dynamic system, giving special attention to damping effects.

Structural elements subjected to in-plane periodic forces Sahu and Datta [5] may lead to dynamic stability, due to certain combinations of the values of load parameters. The instability may occur below the critical load of the structure under compressive loads over a range or ranges of excitation frequencies. kim and choo [2] analyzed dynamic stability of a free- free Timoshenko beam with a concentrated mass .When a pulsating follower force $P_0+P_{1COS}t$ is applied. The discretized equation of motion is obtained by the finite element method, and then the method of multiple scales is adopted to investigate the dynamic instability region.

2.3 TYPES OF PARAMETRIC RESONANCE

Multi degree freedom systems may exhibit simple resonance, resonance of sum type or resonance of difference type depending upon the type of loading, support conditions and system parameter. Mettler [26] furnished a classification for various kinds of resonances exhibited by linear periodic system. Iwatsubo and his co-workers [23-24] from their investigation on stability of columns found that uniform columns with simple supported ends do not exhibit combination type resonances. Saito and Otomi [27] on the basis of their investigation of stability of viscoelastic beams with viscoelastic support concluded that combination resonances of different type do not occur for axial loading, but it exits for tangential type of loading. Celep [18] found that for a simply supported pretwisted column, combination resonances of the sum type may exit or disappear depending on the pretwist angle and rigidity ratio of the cross-section. Ishida et al. [22] showed that an elastic shaft with a disk exhibits only difference type combination resonance. Chen and Ku [19] from their investigations found that for a cantilever shaft disk system, the gyroscopic moment can enlarge the principal regions of dynamic instability.

2.4 EFFECTS OF DAMPING

The destabilizing effect of damping on combination resonances has been observed by many investigators. Iwatsubo [37] found that both internal and external damping degenerate the simple resonance regions of a beam while a predominant external damping destabilizes the combination resonances, internal damping alone may either stabilize or destabilize them.Takahashi [39] generalized the eigen solution method to include systems having different amounts of damping in the various modes. Gurgone [40] found that increasing viscoelasticity stabilizes a simply supported beam under a constant axial end force and subjected to a periodic displacement excitation. Bauchau and Hong [40] formulated a finite element method to analyze the influence of viscous damping on the dynamic response and stability of parametrically excited beams undergoing large deflections and rotations. Mohan Rao [6] studied the effect of Viscoelastic Damping for Noise Control in Automobiles and Commercial Airplanes.

2.5 EFFECTS OF BOLTED JOINTS

Bolted joints are widely used in industries, e.g., pressure vessels, automobiles, machine tools, home appliances and so on. A bolted joint is usually pre-loaded and designed to resist axial or eccentric axial tension as well as shear external loads Ouqi Zhang and Jason [8].

Christian John Hartwigsen [4] conducted experiments on two types of structures one is a beam with the joint in the center; the other is a frame structure with the joint located in one leg. Both structures are subjected to a variety of tests to determine the important effects of the joint. The tests reveal several important effects: 1.Stiffness decreases 2. Damping increases 3.Both damping and stiffness are amplitude-dependent 4.Energy is dissipated in a power law relation with respect to the input force.

Nanda [9] investigated the mechanism of damping and its theoretical evaluation for layered copper cantilever beams jointed with a number of equispaced connecting bolts under an equal tightening torque. He found that intensity of interface pressure, its distribution characteristics, dynamic slip ratio and kinematic coefficient of friction at the interfaces, relative spacing of the connecting bolts, frequency and amplitude of excitation are all found to have an effect on the damping capacity of such structures. This work revealed that the damping capacity of copper structures, jointed with connecting bolts can be improved considerably by increasing the number of layers while maintaining uniform intensity of pressure distribution at the interfaces.

Song et al [7] studied the dynamics of beam structures using adjusted iwan beam element. The adjusted Iwan model consists of a combination of springs and frictional sliders that exhibits non-linear behavior due to the stick–slip characteristic. The beam element developed is two-dimensional and consists of two adjusted Iwan models and maintains the usual complement of degrees of freedom: transverse displacement and rotation at each of the two nodes. Multilayer feed forward neural network scheme was employed to identify the system parameters of the jointed structures.

All structural assemblies have to be joined in some way, by bolting, welding and riveting or by more complicated fastenings such as smart joints. It is known that the added flexibility introduced by the joint to the structure heavily affects its behavior and when subjected to dynamic loading, most of the energy is lost in the joints. Determining the relevant physics of each joint is critical to a validated full body model of the structure. The two most common mechanisms of joint mechanics are frictional slip, and slapping Hamid and Hassan [11]. These mechanisms have their own characteristic features, e.g. frictional slip becomes saturated at very high amplitudes and slapping pushes energy from low to high frequencies.

Bergman and Vakakis [3] used laser vibrometry for identifying the dynamic characteristic of bolted joints. The technique relies on the comparison of the overall dynamics of the bolted structure to that of a similar but unbolted one. The difference in the dynamics of the two systems can be attributed solely to the joint; modeling this difference in the dynamics enables us to construct a non-parametric model for the joint dynamics. Non-contacting, laser vibrometry is utilized to experimentally measure the structural responses with increased accuracy and to perform scans of the structural modes at fixed frequency. Jose Maria Minguez and Jeffrey Vogwell [10] investigated the effect of tightening torques on the life of plates bolted using single a nd double lap joints

Gaul and Lenz [1] performed experiments and theoretically analysed a bolted single-lap joint. The joint was located between two large masses and was excited with axial and torsional sinusoidal forces. Their studies revealed that hysteresis loops measured for different excitation force levels have different slopes (as the force is increased, the slope decreases), indicating a softening system. Their experiments showed two general slopes for the force-deflection curves; they claimed one corresponded to microslip and one to macroslip. They also found that a plot of dissipated energy per cycle vs. displacement yielded two distinct regions, which corresponded to micro- and macro-slip.

2.6 PRESENT WORK

The literature review reveals that investigation of dynamic stability of layered beams with bolted joints has not been done so fare. The main objective of the present work is to carry out theoretical and experimental study to investigate the effect of different system parameters like number of layers and damping factor on the dynamic stability of the layered beams with bolted connections.

The theoretical analysis has been carried out using finite element method in conjunction with Floquet's theory. An experimental set up has been designed and fabricated for the experimental work.

CHAPTER 3

FINITE ELEMENT MODELING

FINITE ELEMENT MODELING

3.1 INTRODUCTION

The Finite Element Method is essentially a product of electronic digital computer age.Though the approach shares many features common to the numerical approximations, it possesses some advantages with the special facilities offered by the high speed computers. In particular, the method can be systematically programmed to accommodate such complex and difficult problems as non homogeneous materials, non linear stress-strain behavior and complicated boundary conditions. It is difficult to accommodate these difficulties in the least square method or Ritz method and etc. an advantage of Finite Element Method is the variety of levels at which we may develop an understanding of technique. The Finite Element Method is applicable to wide range of boundary value problems in engineering. In a boundary value problem, a solution is sought in the region of body, while the boundaries (or edges) of the region the values of the dependant variables (or their derivatives) are prescribed.

Basic ideas of the Finite Element Method were originated from advances in aircraft structural analysis. In 1941 Hrenikoff introduced the so called frame work method, in which a plane elastic medium was represented as collection of bars and beams. The use of piecewise-continuous functions defined over a sub domain to approximate an unknown function can be found in the work of Courant (1943), who used an assemblage of triangular elements and the principle of minimum total potential energy to study the Saint Venant torsion problem. Although certain key features of the Finite Element Method can be found in the work of Hrenikoff (1941) and Courant (1943), its formal presentation was attributed to Argyris and Kelsey (1960) and Turner, Clough, Martin and Topp (1956). The term “Finite Element method” was first used by Clough in 1960.

In early 1960’s, engineers used the method for approximate solution of problems in stress analysis, fluid flow, heat transfer and other areas. A textbook by Argyris in 1955 on Energy Theorems and matrix methods laid a foundation laid a foundation for the development in Finite Element studies. The first book on Finite Element methods by Zienkiewicz and Chung was published in 1967. In the late 1960’s and early 1970’s, Finite Element Analysis (FEA) was applied to non-linear problems and large deformations. Oden’s book on non-linear continua appeared in 1972. Mathematical foundations were laid in the 1970’s.

3.2 BASIC CONCEPT OF FINITE ELEMENT METHOD

The most distinctive feature of the finite element method that separate it from others is the division of a given domain into a set of simple sub domains, called ‘Finite Elements’. Any geometric shape that allows the computation of the solution or its approximation, or provides necessary relations among the values of the solution at selected points called nodes of the sub domain, qualifies as a finite element. Other features of the method include, seeking continuous often polynomial approximations of the solution over each element in terms of nodal values and assembly of element equations by imposing the inter element continuity of the solution and balance of inter element forces.

Exact method provides exact solution to the problem, but the limitation of this method is that all practical problems cannot be solved and even if they can be solved, they may have complex solution.

Approximate Analytical Methods are alternative to the exact methods, in which certain functions are assumed to satisfy the geometric boundary conditions, but not necessary the governing equilibrium equation. These assumed functions, which are simpler, are then solved by any conventional method available. The solutions obtained from these methods have limited range of values of variables for which the approximate solution is nearer to the exact solution.

Numerical Method has been developed to solve almost all types of practical problem. The two common numerical methods used to solve the governing equations of practical problems are numerical integration and finite element technique. Numerical Integration Methods such as Runge-Kutta, Miline’s method etc adopts averaging of slopes of given function at given initial values. These methods yield better solutions over the entire field width but sometimes laborious.

Finite difference method, the differential equations are approximated by finite difference equation. Thus the given governing equation is converted to a set of algebraic equation. These simultaneous equations can be solved by any simple method such as gauss Elimination, Gauss-Seidel iteration method, Crout’s method etc. the method of finite difference yield fairly good results and are relatively easy to program. Hence, they are popular in solving heat transfer and fluid flow problems. However, it is not suitable for problems with awkward irregular geometry and suitable for problems of rapidly changing variables such as stress concentration problems.

Finite element method (F.E.M) has emerged out to be powerful method for all kinds of practical problems. In this method the solution region is considered to be built up of many small-interconnected sub regions, called finite elements. These elements are applied with in interpolation model, which is simplified version of substitute to the governing equation of the material continuum property. The stiffness matrices obtained for these elements are assembled together and boundary conditions of the actual problems are satisfied to obtain the solution all over the body or region FEM is well-suited computer programming.

Boundary element method (B.E.M) like finite element, method is being used in all engineering fields. In this approach, the governing differential equations are transformed into integral identities applicable over the surface or boundary. These integral identities are integrated over the boundary, which is divided into small boundary segments, as infinite element method provided that the boundary conditions are satisfied, set of linear algebraic equations emerges, for which a unique solution is obtained.

3.3 FINITE ELEMENT APPROACHES

There are two differential Finite element approaches to analyze structures, namely

Force method

Displacement method

3.3.1 FORCE METHOD

The number of forces (shear forces, axial forces & bending moments) is the basic unknown in the system of equations

3.3.2 DISPLACEMENT METHOD

The nodal displacement is the basic unknown in the system of equations. The analysis of lever arm plate fixture has been using the concept of finite element method (F.E.M). The fundamental concept of finite element method is that is that a discrete model can approximate any continuous quantity such as temperature, pressure and displacement. There are many problems where analytical solutions are difficult or impossible to obtain. In such cases finite element method provides an approximate and a relatively easy solutions. Finite element method becomes more powerful when combined rapid processing capabilities of computers.

The basic idea of finite element method is to discretize the entire structure into small element. Nodes or grids define each element and the nodes serve as a link between the two elements. Then the continuous quantity is approximated over each element by a polynomial equation. This gives a system of equations, which is solved by using matrix techniques to get the values of the desired quantities.

The basic equation for the Static analysis is:

$$[K]\ [Q] = [F] \tag{3.1}$$

Where [k] = Structural stiffness matrix

[F] = Loads applied

[Q] = Nodal displacement vector

The global stiffness matrix is assembled from the element stiffness matrices. Using these equations the model displacements, the element stresses and strains can be determined.

3.4 ADVANTAGES OF FEM

The advantages of finite element method are listed below:

1. Finite element method is applicable to any field problem: heat transfer, stress analysis, magnetic field and etc.
2. In finite element method there is no geometric restriction. The body or region analyzed may have any shape.
3. Boundary conditions and loading are not restricted. For example, in a stress analysis any portion of the body may be supported, while distributed or concentrated forces may be applied to any other portion.
4. Material properties are not restricted to isotropy and may change from one element to another or even within an element.
5. Components that have different behavior and different mathematical description can be combined together. Thus single finite element model might contain bar, beam, plate, cable and friction elements.
6. A finite element model closely resembles the actual body or region to be analyzed.

7. The approximation is easily improved by grading the mesh so that more elements appear where field gradients are high and more resolution is required.

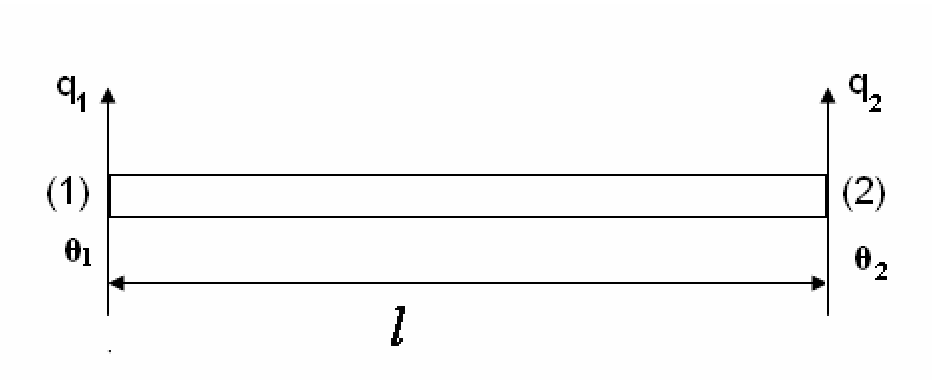
3.5 LIMITATIONS OF FEM

The limitations of finite element method are as given below:

1. To some problems the approximations used do not provide accurate results.
2. For vibration and stability problems the cost of analysis by FEA is prohibitive.
3. Stress values may vary from fine mesh to average mesh.

3.6 ANALYSIS OF TWO NODED BEAM ELEMENT

In the present analysis two noded beam elements with two degrees of freedom (deflection and slope) per node is considered.



q_1 = deflection at node 1

θ_1 = slope at node 1

q_2 = deflection at node 2

θ_2 = slope at node 2

l = length of the element.

The displacement model taken as the polynomial as

$$q = a_1 + a_2 x + a_3 x^2 + a_4 x^3 \tag{3.2}$$

From this displacement model and putting the boundaries equation we arrive.

(1) At x=0; $q = q_1$

$$\therefore \; q_1 = a_1 \tag{3.3}$$

At x=0; $q' = \theta_1$

$$\therefore \quad \theta_1 = a_2 \tag{3.4}$$

(2) At $x = l$; $q = q_2$ and $q' = \theta_2$

$$q_2 = q_1 + \theta_1 l + a_3 l^2 + a_4 l^3 \tag{3.5}$$

$$\theta_2 = \theta_1 + 2 a_3 l + 3 a_4 l^2 \tag{3.6}$$

From equations (3.3) (3.4) (3.5) (3.6) we get

$a_1 = q_1$;

$a_2 = \theta_1$;

$$a_3 = -\frac{3}{l^2} q_1 - \frac{2}{l}\theta_1 + \frac{3}{l^2} q_2 - \frac{\theta_2}{l}$$

$$a_4 = \frac{2}{l^3} q_1 + \frac{\theta_1}{l^2} - \frac{2}{l^3} q_2 + \frac{\theta_2}{l^2}$$

Writing it in a matrix form

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l} \\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2} \end{bmatrix} \begin{Bmatrix} q_1 \\ \theta_1 \\ q_2 \\ \theta_2 \end{Bmatrix}$$

Then,

$$q = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

$$\Longrightarrow \qquad q = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$$

$$\Longrightarrow \qquad q=\begin{bmatrix}1 & x & x^2 & x^3\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -\frac{3}{l^2} & -\frac{2}{l} & \frac{3}{l^2} & -\frac{1}{l}\\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2}\end{bmatrix}\begin{Bmatrix}q_1\\ \theta_1\\ q_2\\ \theta_2\end{Bmatrix}$$

So, $q=[N]\{q\}$ (3.7)

Where, $\{q\}=\begin{Bmatrix}q_1\\ \theta_1\\ q_2\\ \theta_2\end{Bmatrix}$

$[N]$ = Shape function matrix

$[N]=[N_1 \quad N_2 \quad N_3 \quad N_4]$

Where

$$N_1=1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3}$$
$$N_2=x-\frac{2x^2}{l}+\frac{x^3}{l^2}$$
$$N_3=\frac{3x^2}{l^2}-\frac{2x^3}{l^3}$$
$$N_4=-\frac{x^2}{l}+\frac{x^3}{l^2}$$

$$[B]=\frac{\partial}{\partial x^2}[N]$$

Where $[B]$ = strain displacement matrix

$\therefore B=[B_1 \quad B_2 \quad B_3 \quad B_4]$

Where,

$$B_1 = -\frac{6}{l^2} + \frac{12x}{l^3}$$
$$B_2 = -\frac{4}{l} + \frac{6x}{l^2}$$
$$B_3 = \frac{6}{l^2} - \frac{12x}{l^3}$$
$$B_4 = -\frac{2}{l} + \frac{6x}{l^2}$$

(1) Elemental Stiffness matrix:

$$[K] = EI\int_0^l [B]^T [B] dx$$

$$[K] = EI\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \qquad (3.8)$$

(2) Elemental Mass matrix:

$$[M] = \int_0^l [N]^T \rho A [N] dx$$

$$[M] = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \qquad (3.9)$$

Where, A = Cross-sectional area of the element.

ρ = Mass density.

CHAPTER 4

VIBRATION CONTROL
BY DAMPING

VIBRATION CONTROL BY DAMPING

4.1 INTRODUCTION

A method of reducing vibrations in a system is to employ a dynamic vibration absorber. However, when a system is subjected to variable frequency or to broad band random excitation, a number of resonances may be excited. It would then be impracticable to have so many separate vibration absorbers. Some times stiffening of the structures is restored to. This shifts the resonances to higher frequencies and it can be so designed that the excitation frequencies remain lower than the natural frequencies of the structures. However, real vibrating systems are made up by many elements and if one designs one elements natural frequency to be above excitation frequency serious vibration trouble may occur in another element. So in many cases, resonances may be unavoidable. Effort is then directed to reduce amplitude at resonance. This is done by adding damping. Damping reduces resonant response and stresses. It also reduces r.m.s response in random vibrations.

4.2 DAMPING

Damping refers to the extraction of mechanical energy from a vibrating system usually by conversion of this energy into heat. Damping serves to control the steady-state resonant response and to attenuate traveling waves in the structure. There are two types of damping: material damping and system damping. Material damping is the damping inherent in the material while system or structural damping includes the damping at the supports, boundaries, joints, interfaces, etc. in addition to material damping. Various terms such as viscous damping, hysteretic damping, Coulomb damping, linear and proportional damping, etc. are used in the literature to represent vibration damping. It should be noted that these representations merely indicate the mathematical model used to represent the physical mechanism of damping that is still not clearly understood for many cases.

4.3 TYPES OF DAMPING

1. Coulomb (or) dry friction damping
2. Material (or) solid (or) hystertic damping
3. Viscous damping
4. Viscoelastic damping

4.3.1 COULOMB (OR) DRY FRICTION DAMPING

Here the damping force is constant is magnitude but opposite in direction to that of the motion of the vibration body. It is caused by friction between rubbing surfaces that are either dry or have sufficient lubrication.

4.3.2 MATERIAL (OR) SOLID (OR) HYSTERTIC DAMPING

When materials are deformed, energy is absorbed and dissipated by the material. The effect is due to friction between the internal planes, which slip or slide as the deformation take place. When a body having material damping is subjected to vibration, the stress-strain diagram shows a hysteresis loop. The area of this loop denotes the energy lost per unit volume of the body per cycle due to damping.

4.3.3 VISCOUS DAMPING

Viscous damping is the most commonly used damping mechanism in vibration analysis. When mechanical systems vibrate in a fluid medium such as air, gas, water, and oil, the resistance offered by the fluid to the moving body causes energy to be dissipated. In this case, the amount of dissipated energy depends on many factors, such as the size and shape of the vibrating body, the viscosity of the fluid, the frequency of vibration, and the velocity of the vibrating body. In viscous damping, the damping force is proportional to the velocity of the vibrating body.

4.3.4 VISCOELASTIC DAMPING

Viscoelasticity may be defined as material response that exhibits characteristics of both a viscous fluid and an elastic solid. An elastic material such as a spring retracts to its original position when stretched and released, whereas a viscous fluid such as putty retains its extended shape when pulled. A viscoelastic material (VEM) combines these two properties it returns to its original shape after being stressed, but does it slowly enough to oppose the next cycle of vibration. The degree to which a material behaves either viscously or elastically depends mainly on temperature and rate of loading (frequency). Many polymeric materials (plastics, rubbers, acrylics, silicones, vinyl's, adhesives, urethanes, epoxies, etc.) having long chain molecules exhibit viscoelastic behavior. The dynamic properties (shear modulus, extensional modulus, etc.) of linear viscoelastic materials can be represented by the complex modulus approach. The introduction of complex modulus brings about a lot of convenience in studying the material properties of viscoelastic materials. The material properties of viscoelastic materials depend significantly on environmental conditions such as environmental temperature, vibration frequency, pre-load, dynamic load, environmental humidity and so on. Therefore, a good understanding of such effects, both separately and collectively, on the variation of the damping properties is necessary in order to tailor these materials for specific applications.

4.4 FREE VIBRATION WITH COULOMB DAMPING

In many mechanical systems, coulomb (or) dry friction dampers are used because of their mechanical simplicity and convenience. Coulomb damping arises when bodies slide on dry surfaces. Coulomb's law of dry friction states that, when two bodies on dry surfaces. Coulomb's law of dry friction states that, when two bodies are in contact, the force required to produce sliding is proportional to the normal force acting in the plane of contact. Thus the friction force F is given by

$$F = \mu N = \mu W = \mu m g \tag{4.1}$$

Where N is the normal force, equal to the weight of the mass (W=mg) and μ is the coefficient of sliding (or) kinetic friction. The Value of the coefficient of friction (μ) depends on the materials in contact and the condition of the surfaces in contact. For example, $\mu \approx 0.1$ for metal on metal (lubricated) 0.3 for metal on metal (unlubricated), and nearly 1.0 for rubber on metal. The friction force acts in a direction opposite to the direction of velocity. Coulomb damping is some times called constant damping, since the damping force is independent of the displacement and velocity. It depends on the normal force N between the sliding surfaces.

4.5 DAMPING IN STRUCTURAL MEMBERS

Damping inherent in structural members is inadequate; various external damping techniques have been used in practice to enhance their damping capacity.

4.6 DAMPING CAPACITY IMPROVEMENT TECHNIQUES OF STRUCTURAL MEMBERS

1. Application of constrained/unconstrained viscoelastic layers.
2. Sand witch beams.
3. Fabrication of multilayered sandwich construction.
4. Insertion of special high damping inserts.
5. Application of spaced damping techniques.
6. Fabrication of layered construction with either riveted/welded/bolted joint.

Although these techniques are used in actual practice depending on the situation and environmental condition, the last one, i.e., the layered construction jointed with connecting bolts can be used selectively in machine tool structures. Structure with layered and jointed construction using connecting bolts has the flexibility to fabricate the same as to posses' pre-determined damping capacity. However, in such cases, attention has to be focused on proper orientation and spacing of the connecting bolts in order to maximize the damping capacity.

4.7 APPLICATIONS OF LAYERED CONSTRUCTIONL

1. Layered constructions can be used in aerospace applications, machine tool structures etc.
2. Three layered laminated metal/ceramic and ceramic/ceramic Euler–Bernoulli beams are used in the design of high-performance flexural resonators for sensing and communications applications.
3. Two layered construction is used in Turbine blades manufacturing.

4.8 DAMPING IN MACHINE TOOLS

Damping in machine tools basically is derived from two sources--material damping and interfacial slip damping. Material damping is the damping inherent in the materials of which the machine is constructed. The magnitude of material damping is small comparing to the total damping in machine tools. A typical damping ratio value for material damping in machine tools is 0.003. It accounts for approximately 10% of the total damping. The interfacial damping results from the contacting surfaces at bolted joints and sliding joints. This type of damping accounts for approximately 90% of the total damping. Among the two types of joints, sliding joints contribute most of the damping. Welded joints usually provide very small damping which may be neglected when considering damping in joints.

4.9 EFFECT OF DIFFERENT PARAMETERS ON THE DAMPING CAPACITY OF LAYERED STRUCTURES

Several parameters play a major role on the damping capacity of layered structures jointed with connecting bolts. They are: (a) Tightening torque applied to the connecting bolts, (b) amplitude of excitation, (c) frequency of excitation, (d) arrangement of connecting bolts, and (e) number of layers.

CHAPTER 5

THEORETICAL ANALYSIS

THEORETICAL ANALYSIS

5.1 FORMULATION OF THE PROBLEM

Beam with end conditions such as Fixed-Fixed shown in figure (5.1). A typical finite beam element is shown in (5.2) with v_i, θ_i, and v_j θ_j as the nodal degrees of freedom. The matrix equation for vibration of axially loaded discretised system is

$$[M]\left\{\ddot{q}\right\}+[C]\left[\dot{q}\right]+[Ke]\{q\}-P(t)[S]\{q\}=0 \tag{5.1}$$

Where, $\{q\}$= Assemblage nodal displacement vector$\left[v_i,\theta_i,v_j,\theta_j\right]^T$.

The dynamic load P (t) is periodic and can be expressed in the form P = $P_0 + P_t \cos \Omega t$ Where Ω is the disturbing frequency, P_o the static and P_t the amplitude of time dependent component of the load, can be represented as the fraction of the fundamental static buckling load P^*. Hence substituting P = $\alpha P^* + \beta P^* \cos\Omega t$, with α and β as static and dynamic load factors respectively. The equation (1) becomes

$$[M]\left\{\ddot{q}\right\}+[C]\left[\dot{q}\right]+\left([K_e]-\alpha P^*[S_S]-\beta P^*\cos\Omega t[S_t]\right)\{q\}=0 \tag{5.2}$$

Where the matrices $[S_S]$ and $[S_t]$ reflect the influence of P_o and P_t respectively. If the static and time dependent component of loads are applied in the same manner, then $[S_S]$ = $[S_t]$ = [S].

Equation (5.2) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The development of region of instability arises from Floquet's theory which establishes the existence of the periodic solutions of period T and 2T, where T = $\dfrac{2\pi}{\Omega}$.

The boundaries of the primary instability region with period 2T are of practical importance (Bolotin–16) and the solution Eq. (5.2) can be achieved in the form of trigonometric series.

$$q(t)=\sum_{K=1}^{\infty}\left[\{a_k\}\sin\frac{K\theta\, t}{2}+\{b_k\}\sin\frac{K\theta\, t}{2}\right] \tag{5.3}$$

Putting this in equation (5.2) and if only first term of the series is considered, equating coefficients of $\sin\frac{\theta\, t}{2}$ *and* $\cos\frac{\theta\, t}{2}$, the equation becomes.

$$\begin{bmatrix} [K_e]-\alpha P^{*}[S_S]+\frac{\beta}{2}P^{*}[S_t]-\frac{\Omega^2}{4}[M] & -\Omega[C] \\ \Omega[C] & [K_e][S_S]-\alpha \overset{*}{P}[S_S]-\frac{\beta}{2}\overset{*}{P}[S_t]-\frac{\Omega^2}{4}[M] \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}=0 \tag{5.4}$$

Eq. (5.4) can be rearrange as

$$\begin{aligned}&\left\{\left(\frac{\Omega}{2}\right)^2\begin{bmatrix}-[M] & [0] \\ [0] & [M]\end{bmatrix}+\left(\frac{\Omega}{2}\right)\begin{bmatrix}[0] & -2[C] \\ -2C & [0]\end{bmatrix}+\left(\frac{\Omega}{2}\right)\begin{bmatrix}[0] & -2[C] \\ -2C & [0]\end{bmatrix}\right. \\ &\left.+\begin{bmatrix}[K_e]-\alpha \overset{*}{P}[S_S]+\frac{\beta}{2}\overset{*}{P}[S_t] & [0] \\ [0] & [K_e]-\alpha \overset{*}{P}[S_S]-\frac{\beta}{2}\overset{*}{P}[S_t]\end{bmatrix}\right\}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}=0\end{aligned} \tag{5.5}$$

The size of the dynamic stability problem in Eq. (5.5) is double that of the original free vibration problem. To reduce the computational effort the size of the Eq. (5.5) is reduced by means of co-ordinate transformation as follows: Consider the two free vibration problems:

$$\left|[K_e]-\alpha\, P^{*}[S_S]+\frac{\beta}{2}P^{*}[S_t]=\omega_1^2[M]\right|\{q_1\} \tag{5.6a}$$

$$\left|[K_e]-\alpha\, P^{*}[S_S]-\frac{\beta}{2}P^{*}[S_t]=\omega_2^2[M]\right|\{q_2\} \tag{5.6b}$$

Let $[\Lambda_1]$ and $[\Lambda_2]$ be the diagonal matrices containing first m eigenvalues of the problems (5.6a) and (5.6b) respectively in the diagonal elements. Let $[\phi_1]$ and $[\phi_2]$ be in the corresponding $n\times m$ modal matrices.

Let $\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{bmatrix} [\phi_1] & [0] \\ [0] & [\phi_2] \end{bmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \end{Bmatrix}$

Substituting in Eq. (5.5) and premultiplying by

$$\begin{bmatrix} [\phi_1]' & [0] \\ [0] & [\phi_2]' \end{bmatrix}$$

$$\left\{ \left(\frac{\Omega}{2}\right)^2 \begin{bmatrix} -[I] & [0] \\ [0] & [I] \end{bmatrix} + \left(\frac{\Omega}{2}\right) \begin{bmatrix} [0] & -2\left[\hat{C}\right] \\ -2\left[\hat{C}\right]' & [0] \end{bmatrix} + \begin{bmatrix} [\Lambda_1] & [0] \\ [0] & -[\Lambda_2] \end{bmatrix} \right\} \begin{Bmatrix} \xi_1 \\ \xi_2 \end{Bmatrix} = 0$$

where

$$\left[\hat{C}\right] = [\phi_1]' [C][\phi_2]$$

$$\Omega^2 \left[\bar{M}\right]\{\xi\} + \Omega \left[\bar{C}\right]\{\xi\} + \left[\bar{K}\right]\{\xi\} = 0 \qquad (5.7)$$

or

where

$$\left[\bar{M}\right] = \begin{bmatrix} -[I] & [0] \\ [0] & [I] \end{bmatrix}, \left[\bar{C}\right] = 4 \begin{bmatrix} [0] & -\left[\hat{C}\right] \\ -\left[\hat{C}\right]' & [0] \end{bmatrix} \text{ and } \left[\bar{K}\right] = 4 \begin{bmatrix} [\Lambda_1] & [0] \\ [0] & -[\Lambda_2] \end{bmatrix}$$

The solution of free vibration problem can be used to solve Eq. (5.7).Real solutions for Ω correspond to the boundaries of parametric instability regions. The complex solution for Ω means that steady state oscillation can be maintained for any value of the excitation frequency, for that particular mode.

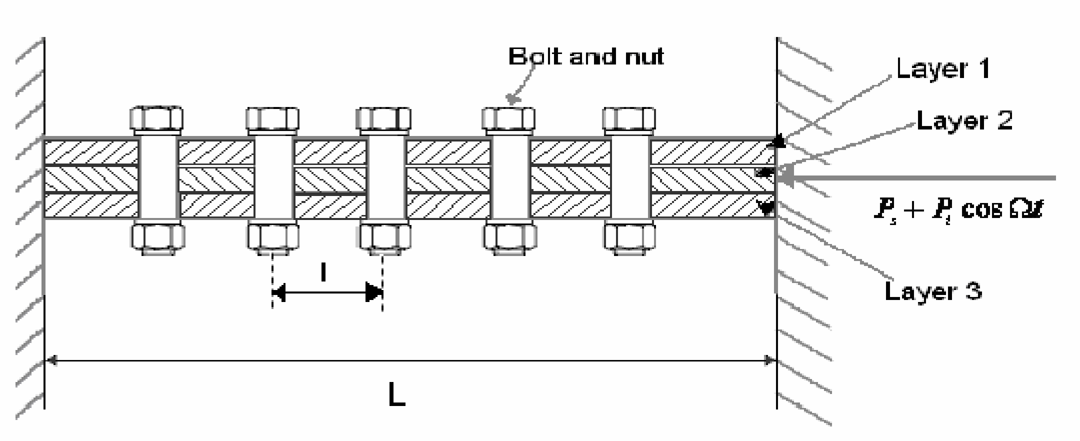


Fig. (5.1) Fixed-fixed three layer beam

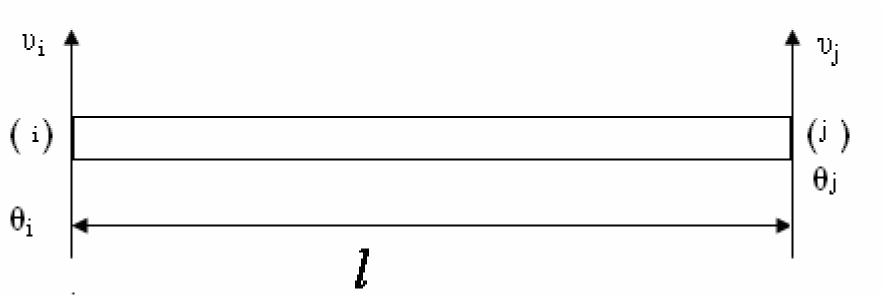


Fig. (5.2) Finite beam element

5.2 ELEMENT MATRICES

The increase in potential energy of an element length '*l*' of a beam subjected to an axial force 'P' is given by

$$U = \frac{1}{2}\int_0^l E(x)\, I(x)\left(\frac{d^2v}{dx^2}\right)^2 dx - \frac{1}{2}P\int_0^l \left(\frac{dv}{dx}\right)^2 dx \tag{5.8}$$

Assuming polynomial expansion for *v* and substituting in equation (5.8) this equation becomes.

$$\text{U} = \frac{1}{2}\{q\}_e^{\,T}[K]\{q\}_e \tag{5.9}$$

Where $[K] = [K_e]_e - P[S]_e$

The kinetic energy T for an elemental length *l* of a beam is given by

$$T = \frac{1}{2}\int_0^l \rho\, A(x)\left\{v'^2\right\}dx \tag{5.10}$$

where ρ is the mass density of the material of the beam using the polynomial expansion for *v* and substituting in equation (5.10) we get

$$\text{T} = \frac{1}{2}\left\{\dot{q}\right\}_e^T [M]_e \left\{\dot{q}\right\}_e^T \tag{5.11}$$

Damping is usually introduced in the dynamic analysis in terms of the energy dissipation in the system, which may be expressed as

$$F = \frac{1}{2}\left\{\dot{q}\right\}^T [C]\left\{\dot{q}\right\} \tag{5.12}$$

The damping Matrix is taken to be the linear combination of mass and elastic stiffness matrix (i.e. damping is assumed to be proportional).

$$[C] = \alpha [M] + \beta [K] \tag{5.13}$$

Where α and β are constants. The constants α and β are determined so as to give the desired damping ratios in the first two modes of vibration of an unloaded beam (P=0). α and β are given by

$$\begin{aligned} \alpha &= \frac{2\left(\omega_2 \xi_2 - \omega_1 \xi_1\right)}{\left(\omega_2^2 - \omega_1^2\right)} \\ \beta &= \frac{2\,\omega_1 \omega_2 \left(\omega_2 \xi_1 - \omega_1 \xi_2\right)}{\left(\omega_2^2 - \omega_1^2\right)} \end{aligned} \tag{5.14}$$

Where ω_1 and ω_2 are first two undamped natural frequencies, and ξ_1 and ξ_2 are the corresponding damping ratios.

The damping factor ξ_1 and ξ_2 are determined experimentally as described in chapter [7]

Element mass matrix, element elastic stiffness matrix and element stability matrix which is a function of the axial load P are given by the expressions

$$[M]_{\mathbf{e}} = \int_0^L [N]^T \rho [N] dx \tag{5.15}$$

$$[K_e]_{\mathbf{e}} = \int_0^L [N^{//}]^T [D][N^{//}] dx \tag{5.16}$$

$$[S]_{\mathbf{e}} = \int_0^L [N^{/}]^T [N^{/}] dx \tag{5.17}$$

Where $[N''] = \frac{\partial^2}{\partial x^2}[N]$, $[N'] = \frac{\partial}{\partial x}[N]$ and [N] is the element shape function matrix

[D]=E(x) I(x).

The overall matrices $[K_e]$, $[S]$ and $[M]$ are obtained by assembling the corresponding element matrices. The displacement vector consists of only active nodal displacements.

5.3 DETERMINATION OF THE MODAL DAMPING FACTOR

The classical method of determining the damping at a resonance, using a frequency analyzer, is to identify the half power (–3 dB) points of the magnitude of the frequency response function see in fig (5.3).For a particular mode, the damping ratio ξ_r can be found from the following equation.

$$\xi_r = \frac{\Delta\omega}{2\omega_n} = \frac{\omega_2 - \omega_1}{\omega_n} \tag{5.18}$$

where $\Delta\omega$ is the frequency bandwidth between the two half power points and ω_n is the resonance frequency.

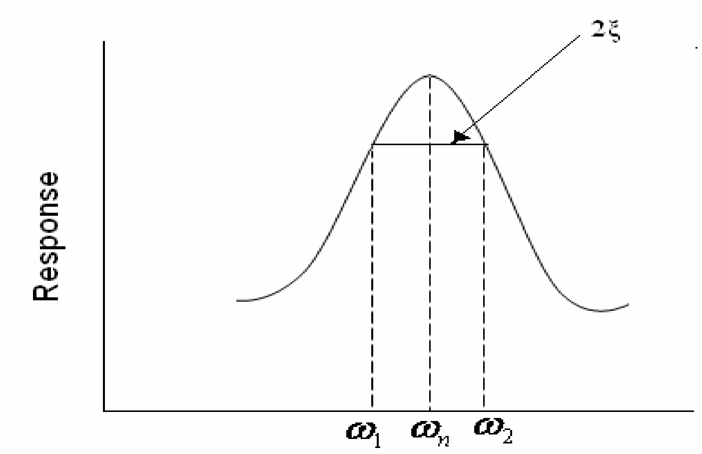


Fig. (5.3) Frequency response

5.4 RESULTS AND DISCUSSION

S.no	No. of layers	Present FEM analysis		Experimental result	
		Ist Mode	2ndMode	Ist Mode	2ndMode
1	2	11.25	22.76	11.065	22.13
2	3	8.63	9.88	8.38	9.63
3	7	7.94	15.5	7.75	15

Table (5.1) Comparison of natural frequency in Hz

To validate the present theoretical finite element model and computational producer the 1st mode and 2nd mode natural frequency calculated from the present analysis was compared with the experimentally obtained values. This comparison is given in table (5.1) from the values it can be seen that the theoretical and experimental values of first two mode natural frequencies are in excellent agreement.

Fig (5.4) shows three instability regions for two layers bolt jointed beam with ξ_1=0.0083, ξ_2=0.0040.

Fig (5.5) shows three instability regions for three layers bolt jointed beam with ξ_1=0.0155, ξ_2=0.0165 and comparing three layers bolt jointed beam with two layers beam. Width of the instability region decreases and the instability region shifts vertically upwards.

Fig (5.6) shows three instability regions for seven layers bolt jointed beam with ξ_1=0.0313, ξ_2=0.0325 and comparing seven layers bolt jointed beam with three and two layers bolt jointed beam .Width of the instability region decreases further .

With increase in number of layers the damping capacity of the beam improves and it results in improvement of the stability of the beam.

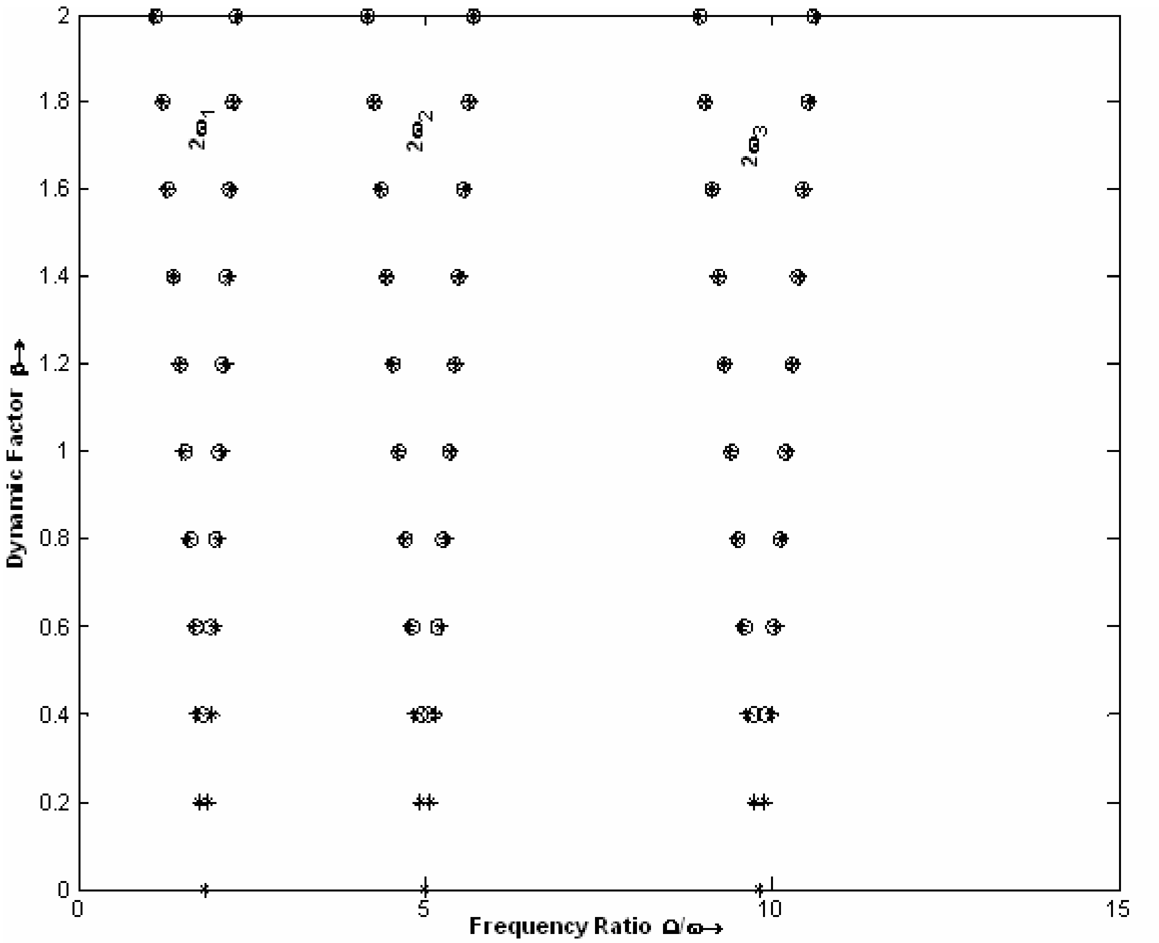


Fig. (5.4), Instability regions, for Two layer case, ξ_1=0.0083, ξ_2=0.0040

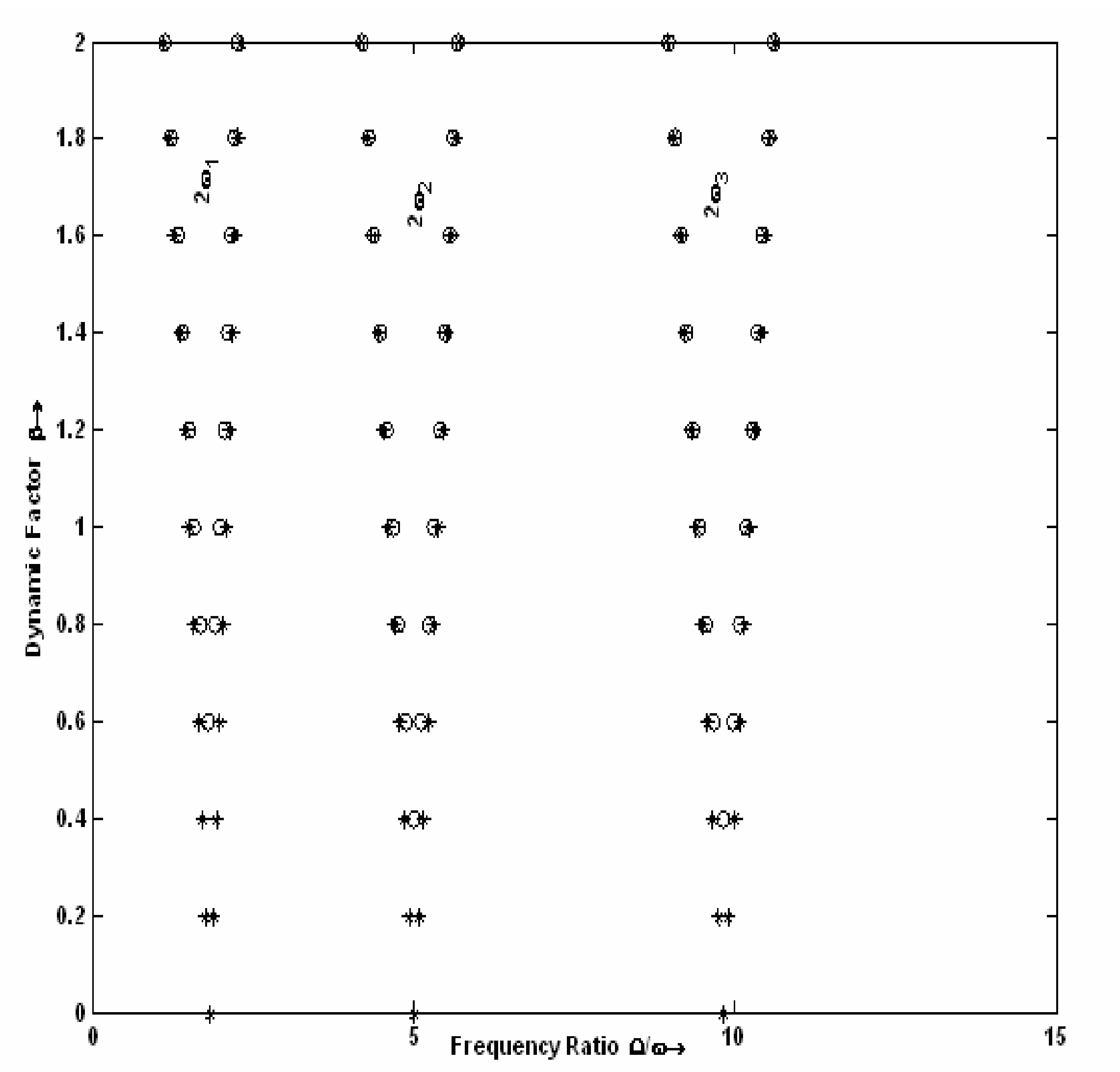


Fig. (5.5), Instability regions, for Three layer case, ξ_1=0.0155, ξ_2=0.0165

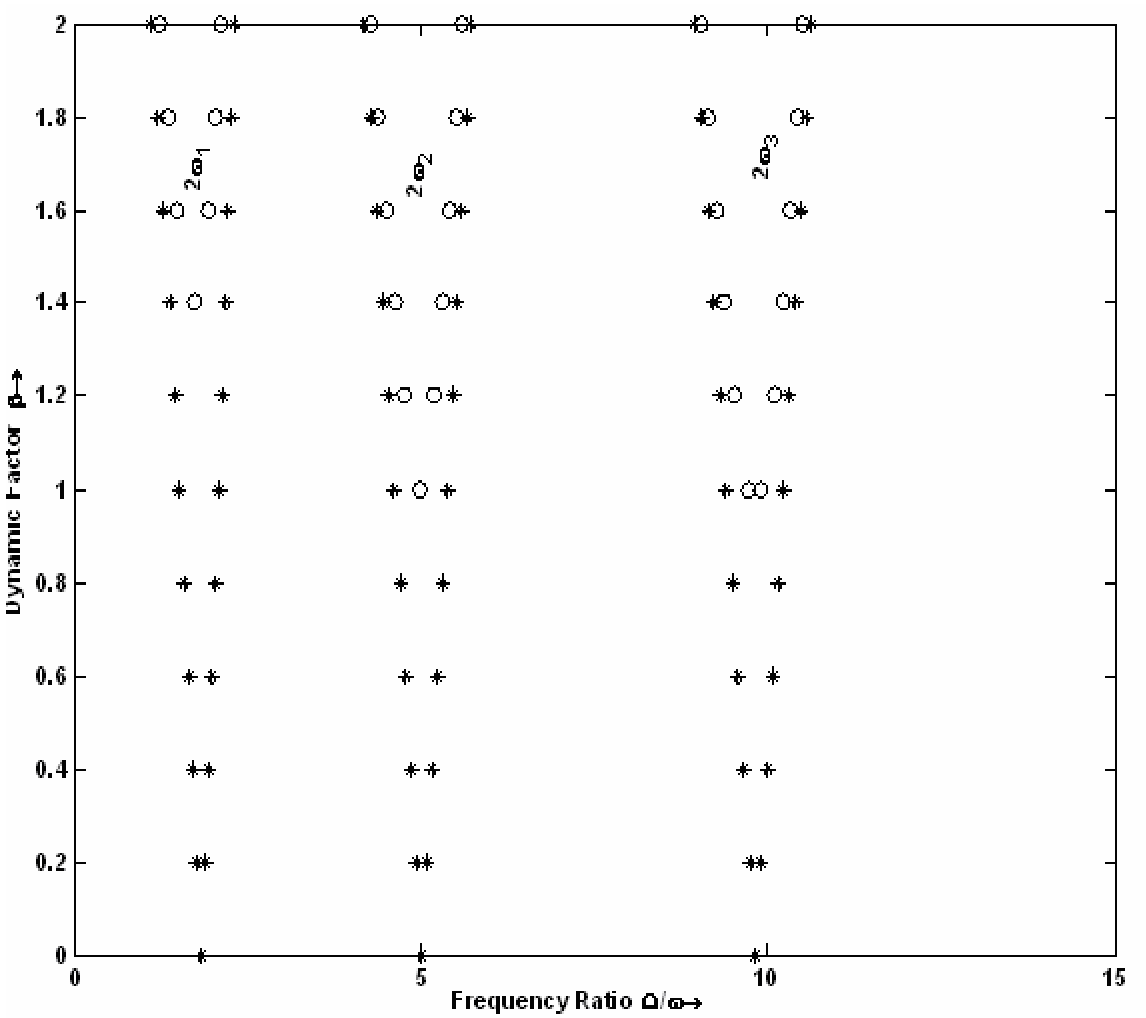


Fig. (5.6), Instability regions, for seven layer case, ξ_1=0.0313, ξ_2=0.0325

CHAPTER 6

EXPERIMENTAL WORK

EXPERIMENTAL WORK

6.1 INTRODUCTION

The purpose of the present experimental work is to validate theoretical results obtained from finite element analysis. Experiments were carried out for beams with two, three and seven layers, keeping the thickness and mass per unit length approximately constant. For this purpose an experimental test rig was designed and fabricated.

6.2 DESCRIPTION OF THE EXPERIMENTAL SET UP

The main components of the test rig are

i. Frame made of mild steel channel section
ii. Beam with screw jack
iii. Electro Dynamic shaker (EDS)
iv. Load cell
v. Power amplifier
vi. Screw jack
vii. Signal Analyzer with pc interface
viii. Laser Vibrometer

The schematic diagram of the equipments used for the experiment and photographic view of the experimental set up are shown in the fig. (6.1) and fig. (6.2) respectively. The set up consists of a framework fabricated from steel channel sections by welding. The frame is fixed in vertical position to the foundation by means of foundation bolts and it has the provision to accommodate beams of different lengths. The periodic axial load $P_t \cos \Omega t$ is applied to the specimen by a 500N capacity electrodynamic shaker (Saraswati Dynamics, India, Model no. SEV-005).

The EDS is placed at the center of concrete foundation. To make the set up flexible/adjustable for specimens of different lengths, some provisions are provided in the frame. Five angles are welded at appropriate locations on inner side of the vertical columns. The screw jack is bolted at the center of the adjustable beam. This beam is fixed on the angles provided according to the requirement i.e. length of the specimen. One of the ends of the specimen is fixed with the screw jack which provides the static force component. The applied load on the specimen is measured by a piezoelectric load cell

(Bruel & Kjaer, model no. 2310-100), which is fixed between the shaker and the specimen. The EDS is connected to the power amplifier/oscillator unit, which is used to operate the EDS at desired frequency and amplitude. The vibration response of the beam specimen is measured through a laser doffler vibrometer.The load cell is placed on the EDS table to record the dynamic load acting on the specimen. The signals from the laser vibrometer and load cell are observed on a computer through a six-channel data acquisition system (B&K, 3560-C), which works on Pulse software platform (B&K 7770, Version 9.0). For fixed end condition the beam end is rigidly clamped to the steel angle bars by means of bolts.

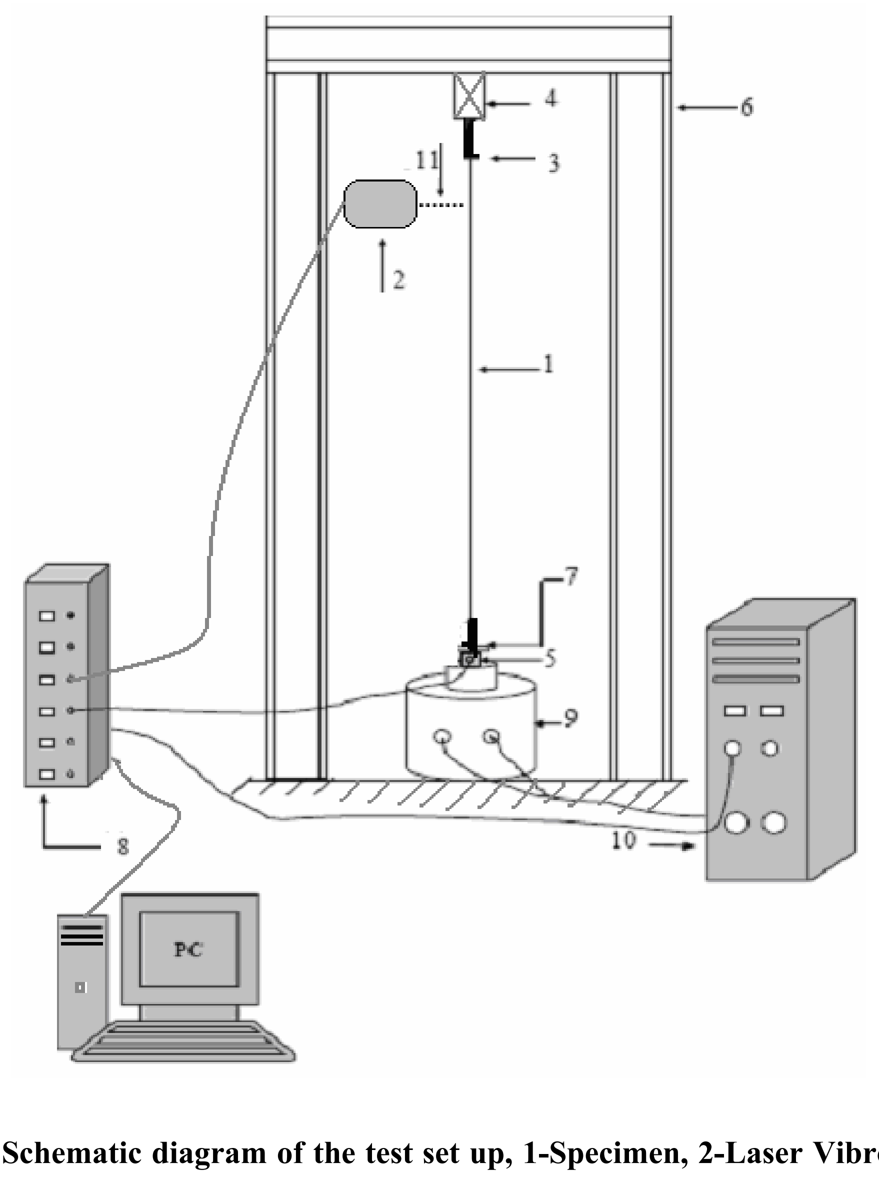


Fig. (6.1) Schematic diagram of the test set up, 1-Specimen, 2-Laser Vibrometer, 3-Upper support (clamped end), 4-Screw jack, 5-Load cell, 6-Frame, 7-Lower support (Clamped end),8- Data acquisition system, 9-Vibration generator, 10-Oscillator and amplifier,11.Laser beam.

Fig. (6.2) Photograph of Experimental Setup

6.3 PREPARATION OF THE SPECIMEN

The specimens were prepared from mild steel strips by joining two (or) more layers using equispaced connecting bolts, which were placed through the joint holes with a washer on either end. With the same tighting torque applied to each bolt. Mass density of the specimen materials was measured by measuring the weight and volume of a piece of specimen material. Each nut was then carefully torqued 6.77 N-m. The bolts were tightened heavily to minimize any interface effects between the bolts, nuts, washers, and beam. The details of the physical and geometric data of the specimens are given in tables (6.1). The schematic diagram of the specimen is shown in fig. (6.3) and photographs of the specimen are shown in fig. (6.4).

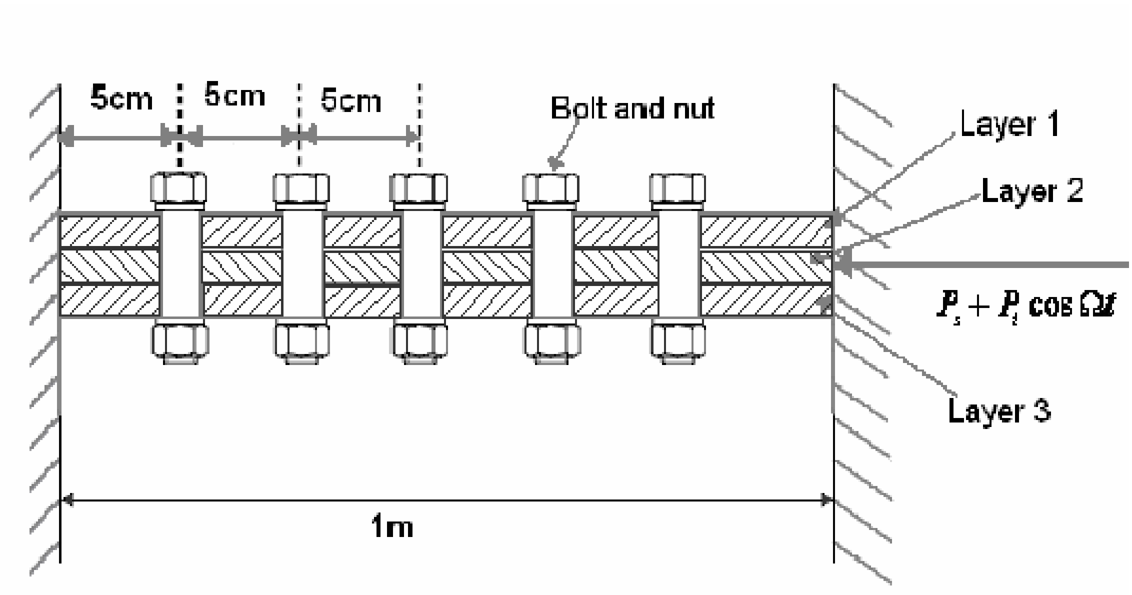


Fig. (6.3) Schematic Diagram of the Specimen

Fig. (6.4) Photograph of Specimen

Fig. (6.5) Photograph of Torque Meter

6.4 DETAILS OF THE SPECIMEN TABLE

Thickness of the specimen ---1.16 mm

Thickness of the single specimen (mm)	Length of the specimen (m)	Diameter of the connecting bolt (mm)	Width of the specimen (mm)	Number of bolted used
1.16	1	6	30	19

Thickness of the multi layered equivalent to the single specimen (mm)	Length of the specimen (m)	Diameter of the connecting bolt (mm)	Width of the specimen (mm)	Number of bolted used	Number of layers used
0.16	1	6	30	19	7
0.32	1	6	30	19	3
0.5	1	6	30	19	2
0.63	1	6	30	19	2

Table (6.1) Details of the specimen

Thickness of the specimen	1.16 mm
Young's Modulus	2.01×10^{11}
Mass density per unit length	4.73352 Kg/m

Table (6.2) Material properties of the specimen

6.5 EXPERIMENTAL PROCEDURE

The experimental has two steps. In the first step the damping factors were experimentally determined for beams with different layers by modal analysis. The specimen was struck with the modal hammer (B&K model 2302-05) to impact an impulse load. The vibration response of the beam and hammer response were recorded on the pc.The damping factors were determined by half power method. In the second step the instability regions were established.

The bolted beam specimen was attached to the screw jack with a specially made clamp and the other end was also clamped to the vibrating table of the Electro Dynamic Shaker. The dynamic load was applied to the specimen by means of Electro Dynamic Shaker. The displacement applied to the specimen is such that the vibration signal is visible on the monitor. The applied displacement can be controlled by means of the amplifier of the shaker. Once the vibration signal is visible the amplitude is kept unchanged. Initially the beam was excited at certain frequency and the amplitude of excitation was increased till the response was observed. Then the amplitude of excitation was kept constant and frequency of excitation was changed in steps of 0.1 Hz.

The vibration signal of the test specimen was recorded by the laser vibrometer. When the vibration response signal suddenly becomes very high (2 to 3 times) the excitation frequency corresponds to the parametric resonance frequency, which is noted down. The frequency of excitation was continuously increased and the frequencies at which the response becomes very high were noted down. These frequencies were divided by the first fundamental frequency (ω) of the system to give the frequency ratio. The dynamic load factor β is calculated by dividing the dynamic load with the fundamental buckling load of the specimen. The unstable boundaries were established experimentally by plotting the points Ω/ω, β and the theoretical and experimental results were compared. The experimental data have been tabulated in tables (7.1-7.3).

6.6 INSTRUMENTS USED IN THE EXPERIMENT

Fig. (6.6) Photograph of Fixed-Fixed end attachment

Fig. (6.7) Photograph of Laser Vibrometer

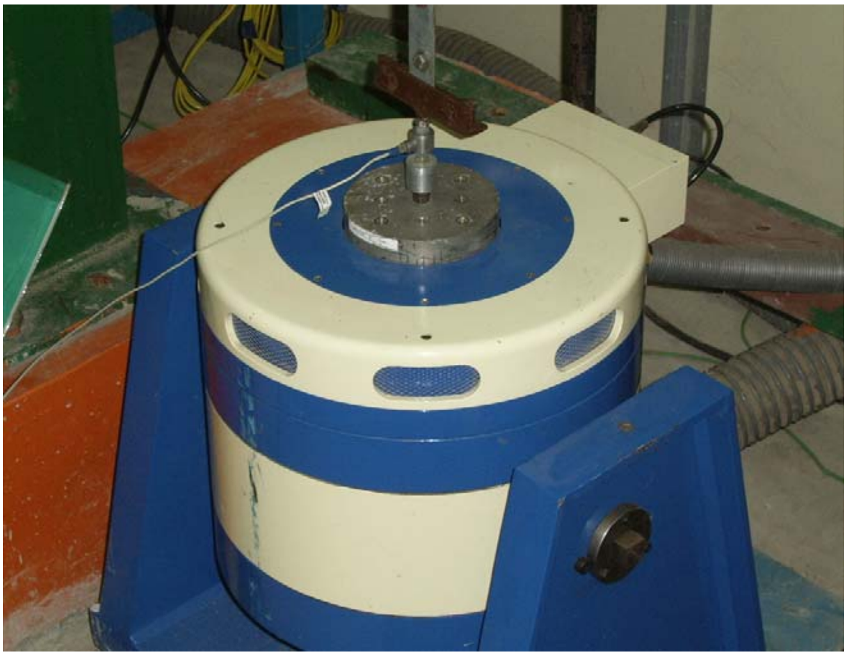


Fig. (6.8) Photograph of Electro Dynamic Shaker

Fig. (6.9) Photograph of Modal Hammer

CHAPTER 7

RESULTS AND DISCUSSION

7. RESULTS AND DISCUSSION

The modal analysis of the beam was carried out to determine the damping factors for the first two modes. The analysis was carried out from the PULSE software platform. The classical method of determining the damping at a resonance, using a frequency analyzer, is to identify the half power (–3 dB) points of the magnitude of the frequency response function. For a first two modes, the damping ratio ξ_1 and ξ_2 can be found from the following graph fig. (7.1-7.3). PULSE type 3560 contains a built-in standard curser reading which calculate the modal damping factor. The specimen was struck with the modal hammer (B&K model 2302-05) to impact an impulse load. The vibration response of the beam and hammer response were recorded on the pc.

Determination of modal frequencies

The resonance frequency is the easiest modal parameter to determine. A resonance is identified as a peak in the magnitude of the frequency response function, and the frequency at which it occurs is found using the analyzer's cursor. By moving the cursor to the peaks of the FRF graph the cursor values and the resonance frequencies were recorded.

Fig (7.1) shows the two layers bolt jointed beam and considering the first two consecutive peaks .By moving the cursor to the first peak of the frequency response function (FRF) graph the resonance frequencies and half power points of the magnitude of the frequencies are noted down and calculate the damping factor ξ_1 and moving the cursor to the second peak of the FRF graph the resonance frequencies and half power points of the magnitude of the frequencies are noted down and calculate the damping factor ξ_2.

Fig (7.2) shows the three layers bolt jointed beam considering the first two consecutive peaks .By moving the cursor to the first peak of the frequency response function (FRF) graph the resonance frequencies and half power points of the magnitude of the frequencies are noted down and calculate the damping factor ξ_1 and moving the cursor to the second peak of the FRF graph the resonance frequencies and half power points of the magnitude of the frequencies are noted down and calculate the damping factor ξ_2.

Fig (7.3) shows the seven layers bolt jointed beam considering the first two consecutive peaks. By moving the cursor to the first peak of the frequency response function (FRF) graph the resonance frequencies and half power points of the magnitude of the frequencies are noted down and calculate the damping factor ξ_1 and moving the cursor to the second peak of the FRF graph the resonance frequencies and half power points of the magnitude of the frequencies are noted down and calculate the damping factor ξ_2.

Graph drawn between the number of layers and damping factors is as shown in fig. (7.4) and fig. (7.5).As the number of layers increase damping factor also increases. As the damping factor increases width of the instability region decreases and stability region increases.

PULSE Report:

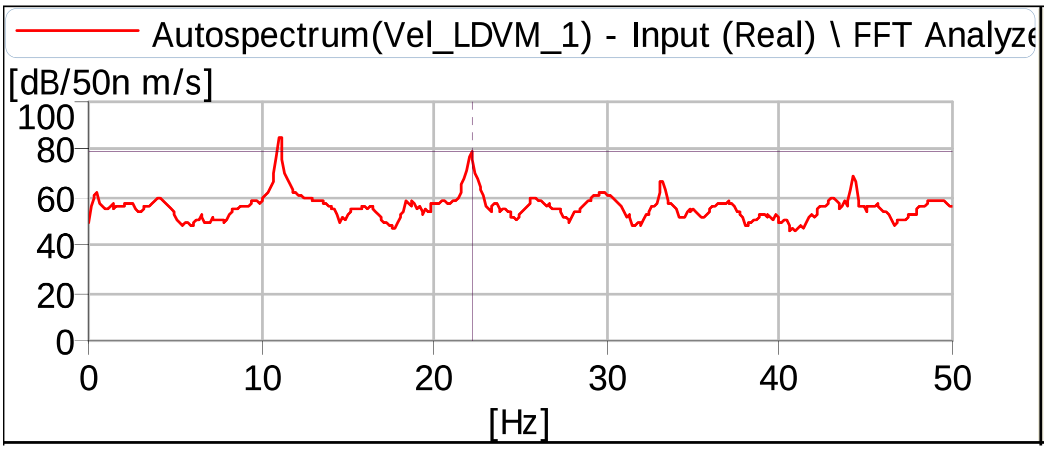


Fig. (7.1) Autospectrum (Vel_LDVM_1) ----Two layers

(From PULSE experiment)

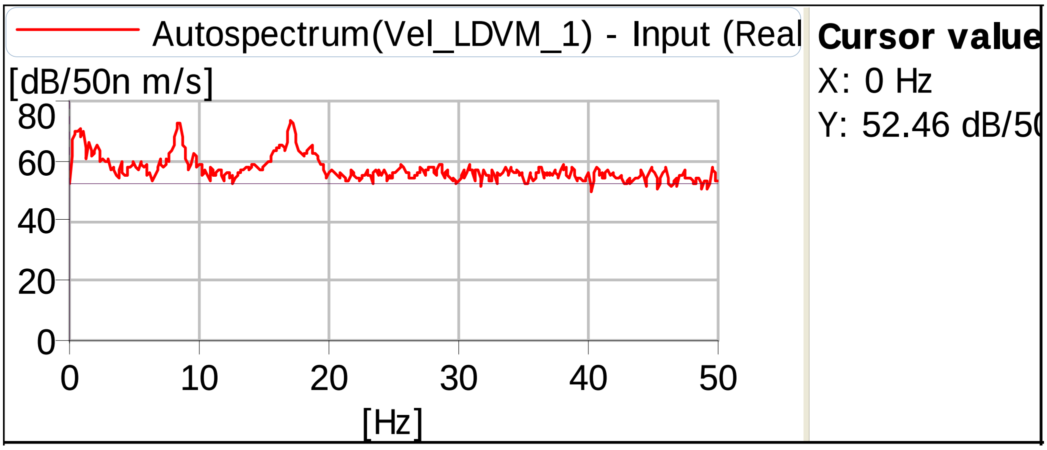


Fig. (7.2) Autospectrum (Vel_LDVM_1) ----Three Layers

(From PULSE experiment)

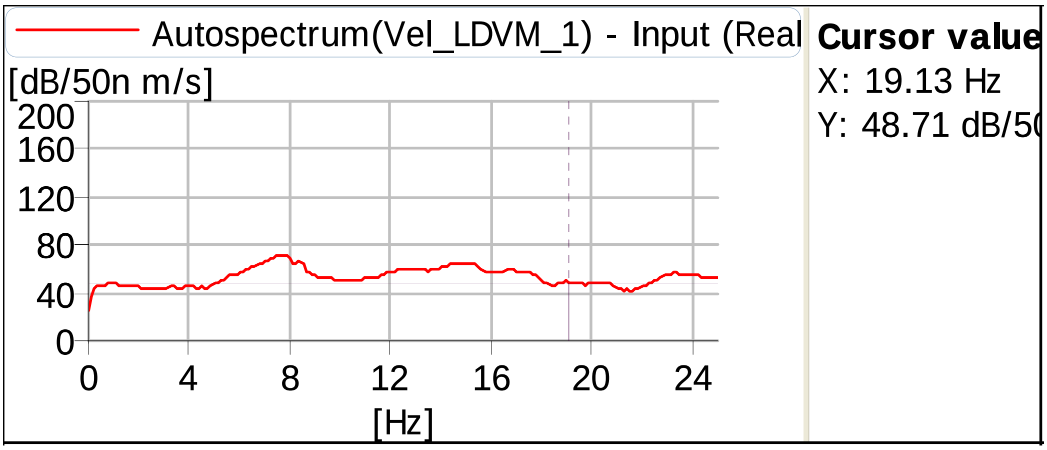


Fig. (7.3) Autospectrum (Vel_LDVM_1) ----Seven Layers

(From PULSE experiment)

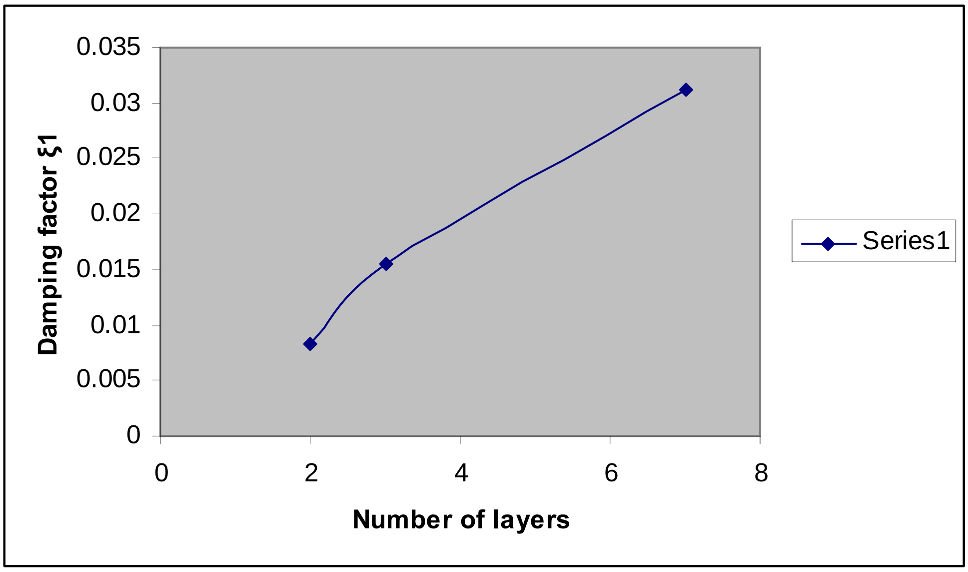


Fig. (7.4) Number of layers vs. damping factor ξ_1

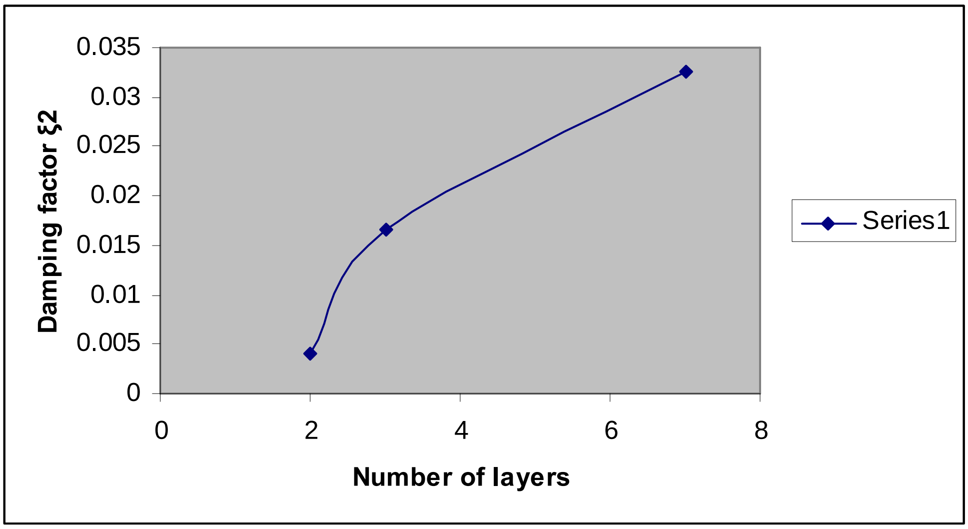


Fig. (7.5) Number of layers vs. damping factor ξ_2

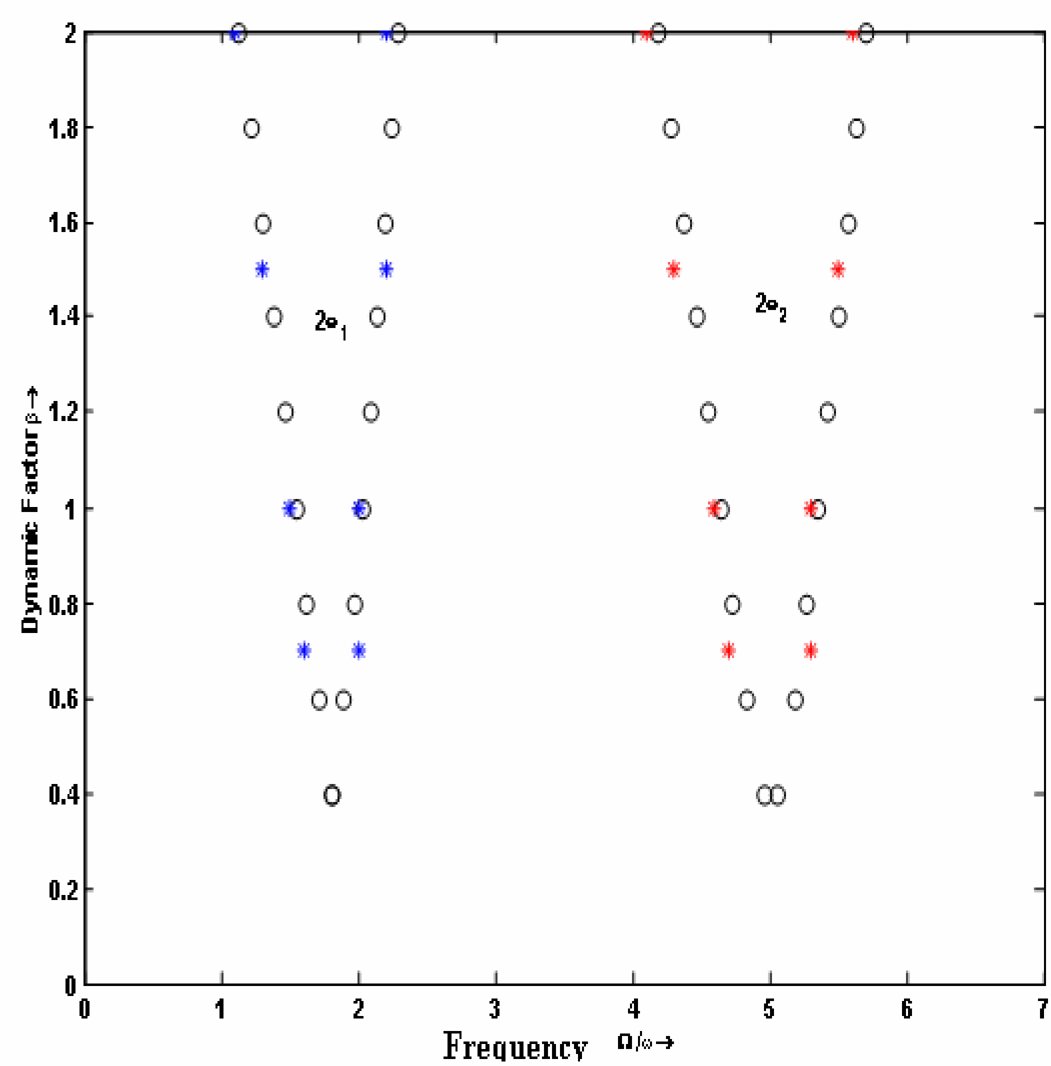


Fig. (7.6) Instability Regions, for Two layer beam, Experimental,' FEM.o

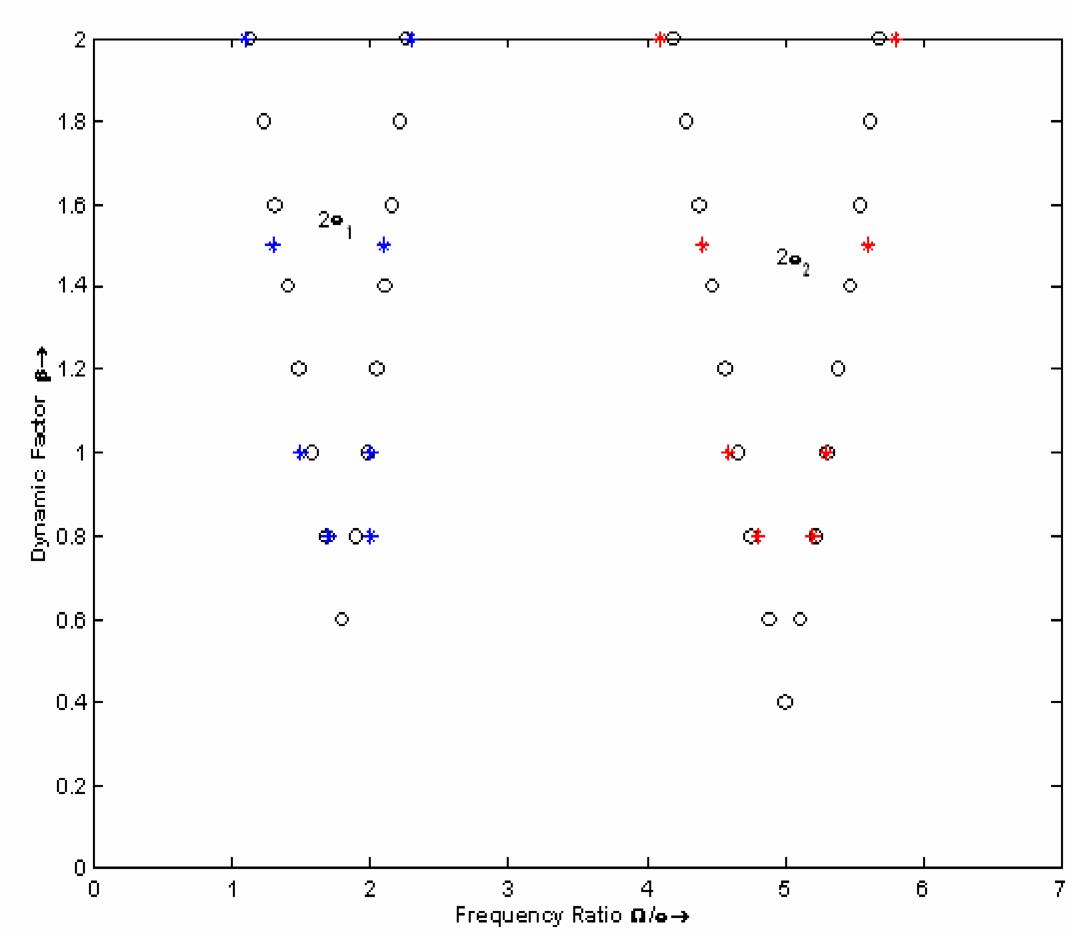


Fig. (7.7) Instability Regions, for Three layer beam, Experimental,' FEM.o

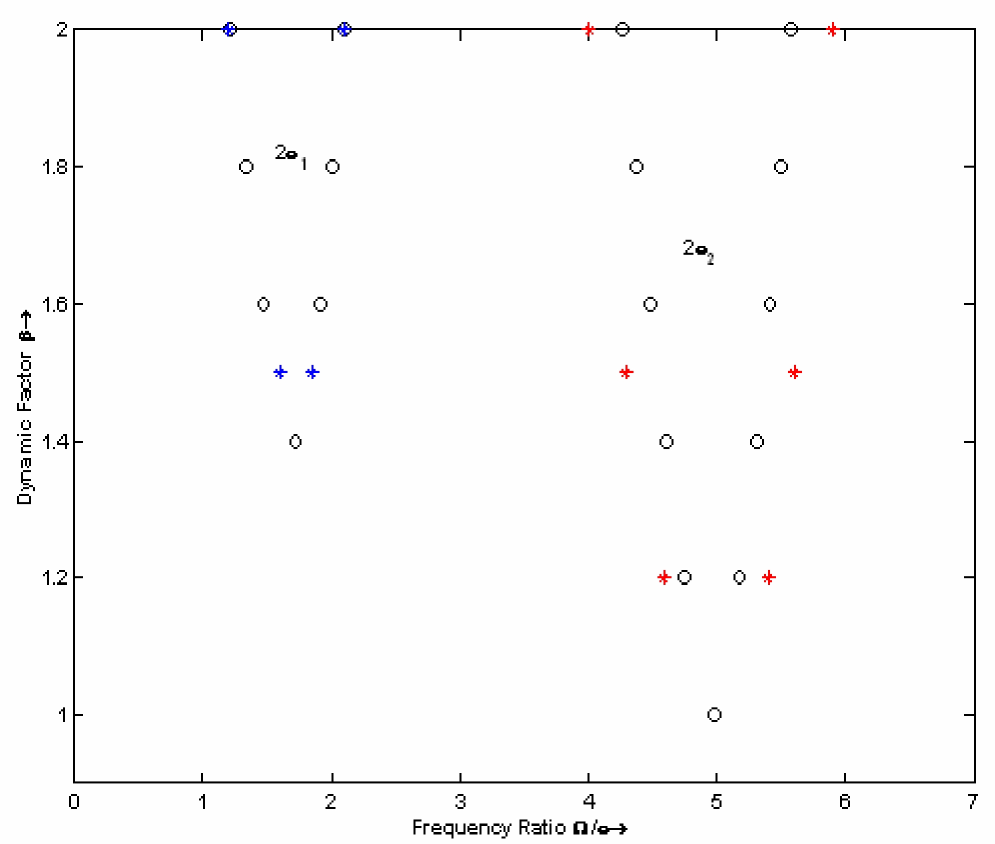


Fig. (7.8) Instability Regions, for Seven layer beam, Experimental,' FEM.o

Experimental boundary frequencies of instability regions

Sl No	Dynamic load Amplitude (P_t) in N	Excitation Frequency (Ω)				Dynamic Load Factor $\beta = P_t / P^*$	Excitation frequency ratio Ω/ω			
		1st Zone		2nd Zone			1st Zone		2nd Zone	
		Lower limit (Ω_{11})	Upper limit (Ω_{12})	Lower limit (Ω_{21})	Upper limit (Ω_{22})		Lower limit (Ω_{11}/ω)	Upper limit (Ω_{12}/ω)	Lower limit (Ω_{21}/ω)	Upper limit (Ω_{22}/ω)
1	14.2	9.8	12.04	28.3	31.9	0.7	1.62	2.0	4.7	5.3
2	20.3	9.0	12.04	27.69	31.9	1.0	1.5	2.0	4.6	5.3
3	30.5	7.8	13.24	25.88	33.11	1.5	1.3	2.2	4.3	5.5
4	40.6	6.6	13.24	24.68	33.71	2.0	1.1	2.2	4.1	5.6

Table 7.1, Experimental boundary frequencies of instability regions for two layer beam P^*=20.3 N, ω=6.02 Hz

Sl No	Dynamic load Amplitude (P_t) in N	Excitation Frequency (Ω)				Dynamic Load Factor $\beta = P_t / P^*$	Excitation frequency ratio Ω/ω			
		1st Zone		2nd Zone			1st Zone		2nd Zone	
		Lower limit (Ω_{11})	Upper limit (Ω_{12})	Lower limit (Ω_{21})	Upper limit (Ω_{22})		Lower limit (Ω_{11}/ω)	Upper limit (Ω_{12}/ω)	Lower limit (Ω_{21}/ω)	Upper limit (Ω_{22}/ω)
1	16.24	10.2	12.04	28.9	31.3	0.8	1.7	2.0	4.8	5.2
2	20.3	9.03	12.04	27.7	31.9	1.0	1.5	2.0	4.6	5.3
3	30.45	7.82	12.64	26.4	33.7	1.5	1.3	2.1	4.4	5.6
4	40.6	6.62	13.84	24.6	34.9	2.0	1.1	2.3	4.1	5.8

Table 7.2., Experimental boundary frequencies of instability regions for three layer beam P^*=20.3 N, ω=6.02 Hz.

Sl No	Dynamic load Amplitude (P_t) in N	Excitation Frequency (Ω)				Dynamic Load Factor $\beta = P_t / P^*$	Excitation frequency ratio Ω/ω			
		1st Zone		2nd Zone			1st Zone		2nd Zone	
		Lower limit (Ω_{11})	Upper limit (Ω_{12})	Lower limit (Ω_{21})	Upper limit (Ω_{22})		Lower limit (Ω_{11}/ω)	Upper limit (Ω_{12}/ω)	Lower limit (Ω_{21}/ω)	Upper limit (Ω_{22}/ω)
1	12.36	-	-	27.7	32.5	1.2	-	-	4.6	5.4
2	30.45	9.63	11.13	25.8	33.7	1.5	1.6	1.85	4.3	5.6
3	40.6	7.22	12.6	24.08	35.5	2.0	1.2	2.1	4.0	5.9

Table 7.3, Experimental boundary frequencies of instability regions for seven layer beam P^*=20.3 N, ω=6.02 Hz.

CHAPTER 8

CONCLUSION AND SCOPE FOR FUTURE WORK

CONCLUSION AND SCOPE FOR FUTURE WORK

8.1 CONCLUSION

The results from a study of dynamic stability of jointed beam subjected to pulsating follower force and the effect of damping on the dynamic stability behavior can be summarized as follows.

1. The theoretical and experimental investigation shows that the multilayered bolt jointed beam with fixed-fixed end conditions is more stable than single bolt layered beams with similar end condition.
2. As the number of layers increases area of the instability region decreases.
3. As the number of layer increases damping factor also increases.
4. Damping generally reduces the width of the instability regions.
5. With increase in number of layers the instability region shift vertically upward.

8.2 SCOPE FOR FUTURE WORK

1. Optimization of different parameters like thickness of layers, spacing of bolts, tighting torque etc.
2. The present analysis can be extended to plates with bolted joints

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